Should the Examiner believe that anything further would be desirable in order to place this application in even better condition for allowance, the Examiner is invited to contact the undersigned at the telephone number set forth below.

Respectfully submitted,

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WPB:SQL/rle

Attachment:
Crawford, Frank S., Ed., Berkeley Physics Course, Volume 3, "Waves," McGraw-Hill, New York, 1968, pages 578-579.

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We see that for the purely resistive frequency range $0 \leqq \omega \ll \Gamma$, the conductivity $\sigma(\omega)$ is real and is equal to its DC (zero frequency) value $\sigma(0)$. The velocity $\dot{x}$ is then in phase with E_x.

The complex electric susceptibility $\chi(\omega)$ is pure imaginary for $\omega \ll \Gamma$, according to Eq. (98). The complex square of the index of refraction, n^2, is then given for $\omega \ll \Gamma$ by

$$n^2 = 1 + 4\pi\chi = 1 + i\frac{4\pi Nq^2}{M}\frac{1}{\omega\Gamma} = 1 + i\frac{\omega_p^2}{\omega\Gamma}, \tag{100}$$

where

$$\omega_p^2 \equiv \frac{4\pi Nq^2}{M}. \tag{101}$$

There are two limiting cases of a "purely resistive medium" with qualitatively different physical characteristics.

Case 1: "Dilute resistive medium"

That means ω, Γ, and ω_p satisfy the relations

$$\omega_p \ll \Gamma, \qquad \frac{\omega_p^2}{\Gamma} \ll \omega \ll \Gamma. \tag{102}$$

Then according to Eq. (100) we have

$$n = \left[1 + i\frac{\omega_p^2}{\omega\Gamma}\right]^{1/2} \approx 1 + \frac{1}{2}i\frac{\omega_p^2}{\omega\Gamma}, \tag{103}$$

with neglect of higher-order terms. Then

$$k = n\frac{\omega}{c} = \frac{\omega}{c} + i\frac{1}{2}\frac{\omega_p^2}{c\Gamma} = \frac{\omega}{c} + \frac{2\pi i}{c}\sigma_{DC}, \tag{104}$$

where we used Eqs. (101) and (93) in the last equality. The real part of k is equal to ω/c, just as in vacuum. The imaginary part is much smaller than the real part. The imaginary part of k gives an exponential attenuation of a traveling plane wave. The mean attenuation length is large compared with one wavelength. The *intensity* of the plane wave is proportional to the absolute square of the complex amplitude. Therefore it is exponentially attenuated with distance by a factor $\exp(-2k_I z)$, where k_I is the imaginary part of k. The distance $d \equiv (2k_I)^{-1}$ in which the *intensity* is attenuated by a factor e^{-1} is given by Eq. (104):

$$\frac{1}{d} \equiv 2k_I = \frac{4\pi}{c}\sigma_{DC}, \qquad \text{i.e.,}\ \frac{\rho_{DC}}{d} = \frac{4\pi}{c}. \tag{105}$$

The "resistance per square" of a square slab of dilute resistive medium having slab thickness d and edge lengths L is the DC resistivity divided by d.

That equals $4\pi/c = 377$ ohms per square, according to Eq. (105). You may recall that 377 ohms per square is the characteristic impedance for "perfect termination" of an electromagnetic plane wave. (See Chap. 5.) Of course the wave is not absorbed in just one exponential decay length d of the intensity. However practically none is reflected, and it is all eventually absorbed.

More precisely, since the real part of n is essentially unity and the imaginary part is small compared with unity, the fractional reflected intensity for a plane wave normally incident from vacuum is given by

$$|R|^2 = \frac{(n_R - 1)^2 + n_I^2}{(n_R + 1)^2 + n_I^2} \approx \frac{0 + n_I^2}{2^2 + n_I^2} \approx \frac{n_I^2}{4} \ll 1. \tag{106}$$

Using Eqs. (103) and (105) this becomes

$$|R|^2 \approx \frac{1}{16}\left(\frac{\omega_p^2}{\omega\Gamma}\right)^2 = \left(\frac{\lambda\!\!\!^{-}}{4d}\right)^2 \ll 1, \tag{107}$$

where $\lambda\!\!\!^{-} \equiv c/\omega$ is the "reduced" wavelength in vacuum.

Case 2: "Dense resistive medium"

That means we have the relations

$$\omega \ll \Gamma, \qquad \omega \ll \omega_p, \qquad \omega\Gamma \ll \omega_p^2. \tag{108}$$

Then n^2 is essentially pure imaginary, according to Eq. (100). When we take the square root of n^2, we use the fact that the square root of i is $[\exp(i\tfrac{1}{2}\pi)]^{1/2} = \exp(i\tfrac{1}{4}\pi)$, which is $2^{-(1/2)}(1 + i)$. That gives

$$n = \left[i\frac{\omega_p^2}{\omega\Gamma}\right]^{1/2} = \left(\frac{\omega_p^2}{2\omega\Gamma}\right)^{1/2}(1 + i) = |n|\frac{(1 + i)}{\sqrt{2}} \tag{109}$$

Then

$$\begin{aligned} k = n\frac{\omega}{c} &= \sqrt{\frac{\omega}{c}}\left(\frac{\omega_p^2}{2c\Gamma}\right)^{1/2}(1 + i) \\ &= \sqrt{\frac{\omega}{c}}\left(\frac{2\pi\sigma_{DC}}{c}\right)^{1/2}(1 + i). \end{aligned} \tag{110}$$

Then the real and imaginary parts of k are equal. Each is large compared to the vacuum value of k (that is, ω/c). The mean penetration distance for the amplitude, k_I^{-1}, is small compared with the vacuum wavelength. It then turns out that a plane wave incident from vacuum to a dense resistive medium is reflected with practically no absorption. That is because the penetration distance is so small that relatively few charges feel any electric field. Those which *do* feel it are at terminal velocity, in phase with E_x, and hence are absorbing energy. However they are so few that the wave "escapes" with little loss of intensity.